Topic 1 // Lesson 05

{ To determine an election using *Pairwise Comparisons*

- Like a Round Robin Tournament in which every player plays once against every other, where *head-to-head* matches are called **pairwise comparisons**.

Method of Pairwise Comparions

- Each candidate goes head-to-head and winner gets a single point
- If a tie, then both get 1/2 point
- Winner is candidate with most points
- Ties are very common in this method

Method of Pairwise Comparions

# of voters	14	10	8	4	1
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

A vs B: 14 to 23, B=1

A vs C: 14 to 23, C=1

A vs D: 14 to 23, D=1

B vs C: 18 to 19, C=1

B vs D: 28 to 9, B=1

C vs D: 25 to 12, C=1

Overall Tally: A = **0** B= **2** C= **3** D= **1**

The Winner is: **Person C**

Method of Pairwise Comparions

# of voters	2	6	4	1	1	4	4
1st	A	B	B	C	C	D	E
2nd	D	A	A	B	D	A	C
3rd	C	C	D	A	A	E	D
4th	B	D	E	D	B	C	B
5th	E	E	C	E	E	B	A

Overall Tally: A = B= C= D=

The Winner is:

Ex 1.12 LAXer's Draft Choice Election

# of voters	2	6	4	1	1	4	4
1st	A	B	B	B	D	D	E
2nd	D	A	A	A	A	A	D
3rd	B	D	D	D	B	E	B
4th	E	E	E	E	E	B	A

Overall Tally: A = B= C= D=

The Winner is:

Ex 1.12 LAXer's Draft Choice Election

If Candidate X is a winner of an election and in a recount one of the *nonwinning* candidates withdraws or is disqualified, then X should still be a winner of the election.

Alternative Interpretation: If Candidate X is a winner of an election and in a reelection another candidate that has no chance of winning (an "irrelevant alternative") enters the race, then X should still be the winner

Independence of Irrelevant Alternatives Criterion (IIA)

Add up 1 to 99.

How many Pairwise Comparisons?

Gauss

How many Pairwise Comparisons?

Sum of Consecutive Integers Formula

$$1 + 2 + 3 + \ldots + L = \frac{L(L + 1)}{2}$$

Counting Pairwise Comparisons

Consider election with 10 candidates:

A, B, C, D, E, F, G, H, I, and J

Compare A with 9 others is 9 comparisons

Compare B with 8 others is 8 comparisons

Compare C with 7 others is 7 comparisons

Compare D with 6 others is 6 comparisons

Continuing until J…

Total #: 1 + 2 + 3 + ..+ 8 + 9 =

Number of Pairwise Comparisons

In an election with N candidates the total number of pairwise comparisons is

$$\frac{(N - 1)N}{2}$$

Practice 1. How many comparisons would occur in a 12 candidate race?
Practice 2. How many comparisons would occur in a 20 candidate race?

In Conclusion

1. Elections are more than just for president and governors. It is for deciding where to eat, getting a job, etc.
2. There are many different methods to vote
3. Outcomes can change with different voting strategies
4. Elections should be fair.

Fairness Criterions

Majority Criterion: A majority candidate should always win the election

Condorcet Criterion: A Condorcet Candidate (winner of head-to-head) should always win the election.

Fairness Criterions

Monotonicity Criterion: If candidate X wins an election, then a second election where that candidate gains votes should still win the election.

Independence of Irrelevant Alternatives (IIA): If candidate X wins an election, then a second election where a candidate exits or irrelevant candidates enters the race, then Candidate X should still win.

A fair voting method should clearly satisfy all four of the above

Plurality violates Condorcet and IIA Criterions. Borda Count violates the Majority, Condorcet and IIA Criterions. Plurality w/ Elimination violates the Condorcet, Monotonocity, and IIA Criterion. Pairwise Comparisons violates the IIA Criterion

Arrows Impossibility Theorem

It is mathematically impossible for a democratic voting method to satisfy all of the fairness criteria.